

## Practice problem

Jackson learned calculus in the 9th grade. Because his brain is suffused with advanced mathematics, his head of radius  $a$  can be modeled as a pressure-release sphere. Meanwhile, his iconic clear-frame glasses, which sit at  $\theta = 45^\circ$ , impose band of rigidity around his head. The boundary conditions for Jackson's head are therefore

$$p(a, \theta) = \begin{cases} 0 & \text{for } \theta \neq \pi/4 \\ p_0 & \text{for } \theta = \pi/4 \end{cases}$$

On the other hand, Chirag did not study calculus until the 12th grade. As such, the top half of his head can be modeled be as a hemisphere with a rigid boundary. The bottom half of his head is pressure-release. The boundary conditions for Chirag's head are therefore

$$p(a, \theta) = \begin{cases} p_0 & \text{for } 0 \leq \theta \leq \pi/2 \\ 0 & \text{for } \pi/2 < \theta < \pi \end{cases}$$

You are given the orthogonality integral

$$\int_{-1}^1 P_n(z)P_m(z) dz = \frac{2}{2n+1} \delta_{nm}$$

as well as the integral

$$\int_0^{\theta_0} P_m(\cos \theta) \sin \theta d\theta = \frac{1}{2m+1} (P_{m-1}(\cos \theta_0) - P_{m+1}(\cos \theta_0))$$

1. Solve the pressure wave equation for Jackson's head.
2. Solve the pressure wave equation for Chirag's head.