Angle of Intromission

Perfect transmission occurs when the acoustic impedances of two media are equal:

$$\begin{aligned} Z_{\text{ac}2} &= Z_{\text{ac}1} \\ Z_2/S_2 &= Z_1/S_1 \\ Z_2/\cos\theta_t &= Z_1/\cos\theta_i \end{aligned}$$

Solving the above for θ_t,

$$\cos\theta_t = \frac{Z_2}{Z_1}\cos\theta_i \tag{1}$$

Meanwhile, Snell's law reads

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i \tag{2}$$

Squaring equation (1)

$$\cos^2\theta_t = \frac{Z_2^2}{Z_1^2}\cos^2\theta_i \tag{3}$$

Squaring equation (2),

$$\sin^2\theta_t = \frac{c_2^2}{c_1^2}\sin^2\theta_i \tag{4}$$

Adding equations (3) and (4),

$$\begin{aligned} 1 &= \frac{c_2^2}{c_1^2}\sin^2\theta_i + \frac{Z_2^2}{Z_1^2}\cos^2\theta_i \\ &= \frac{c_2^2}{c_1^2}\sin^2\theta_i + \frac{Z_2^2}{Z_1^2}\left(1 - \sin^2\theta_i\right) \end{aligned}$$

Solving the above for $\sin^2\theta_i$,

$$\sin^2\theta_i = \frac{1 - Z_2^2/Z_1^2}{c_2^2/c_1^2 - Z_2^2/Z_1^2}$$

Multiplying the numerator and denominator by $Z_1^2/Z_2^2 = \rho_1^2 c_1^2/\rho_2^2 c_2^2$,

$$\sin^2\theta_i = \frac{(Z_1/Z_2)^2 - 1}{(\rho_1/\rho_2)^2 - 1} \tag{B-14}$$