$A\vec{x} = \vec{b}$: Some general tensorial linear equation.

One representation in the basis $\vec{x}'$:

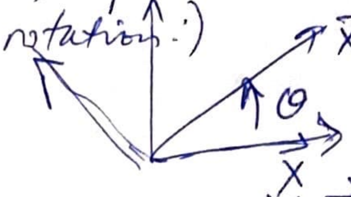
$$A'\vec{x}' = \vec{b}' . \quad \text{......} \quad (1)$$

Another representation in the basis $\vec{x}$:

$$A\vec{x} = \vec{b} . \quad \text{.....} \quad (2).$$

The goal is to relate A' and A (The two different matrix representations of A, the tensor) given the mapping

$S\vec{x}' = \vec{x}$. (for example a rotation.)



↓

Invert: $\vec{x}' = S^{-1}\vec{x}$. (3)

Multiplying (1) by S and noting that $S\vec{b}' \hat{=} \vec{b}$ gives

$$SA'\vec{x}' = S\vec{b}' = \vec{b}.$$

Invoking (3) gives

$$SA'S^{-1}\vec{x} = \vec{b}.$$

Comparing to eq (2), $A\vec{x} = \vec{b}$ shows that

This is what was desired $\boxed{A = SA'S^{-1}}$ ← multiplying by S^{-1} on Right & S on left gives equivalently $A' = S^{-1}AS$.

Alternatively, Start with ~~$S^{-1}\vec{x} = \vec{x}'$~~ $\hat{S}\vec{x}' \hat{=} \vec{x}$ and eq. (2).

$$A\vec{x} = \vec{b}$$

$$AS\vec{x}' = \vec{b}$$

Note that $\vec{b} = S\vec{b}' \to S^{-1}\vec{b} = \vec{b}'$

$$S^{-1}AS\vec{x}' = S^{-1}\vec{b} = \vec{b}'$$

Comparing to (1), $A'\vec{x}' = \vec{b}'$ shows that

$$\boxed{S^{-1}AS = A'}$$

✓ i.e., the inverse mapping