$$\begin{pmatrix} 2 & 0 & -2 \\ -2i & i & 2i \\ 1 & 0 & -1 \end{pmatrix} \to \det \begin{pmatrix} 2-\lambda & 0 & -2 \\ -2i & i-\lambda & 2i \\ 1 & 0 & -1-\lambda \end{pmatrix} = 0.$$

$$(2-\lambda)[(i-\lambda)(-1-\lambda)] + 2(i-\lambda) = 0.$$

$$(2-\lambda)[-i - i\lambda + \lambda + \lambda^2] + 2i - 2\lambda = 0.$$

$$-\cancel{2i} - 2i\lambda + \cancel{2\lambda} + 2\lambda^2 + i\lambda + i\lambda^2 - \lambda^2 - \lambda^3 + \cancel{2i} - \cancel{2\lambda} = 0.$$

$-\lambda^3 + \lambda^2(1+i) + \lambda(-i) = 0$; multiply by (-1).

$$\lambda^3 - \lambda^2(1+i) + \lambda i = 0. \qquad \boxed{\lambda_1 = 0}$$

$$\lambda^2 - \lambda(1+i) + i = 0.$$

$$\lambda = [1 + i \pm \sqrt{(1+i)^2 - 4i}\,]/2$$
$$= [1 + i \pm \sqrt{1 + 2i - 1 - 4i}\,]/2$$
$$= [1 + i \pm \sqrt{-2i}\,]/2.$$

$$(-2i)^{1/2} = [2e^{i\,\mathrm{atan}\,-\infty}]^{1/2} = \sqrt{2}\,(e^{-i\pi/2})^{1/2}.$$
$$= \sqrt{2}\, e^{-i\pi/4} = \sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]$$
$$= \sqrt{2}\left[\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right]$$
$$= 1 - i.$$

$$\lambda = \frac{1 + i \pm [1-i]}{2}$$

$$= \begin{cases} 2/2 = \boxed{1 = \lambda_2} \\ 2i/2 = \boxed{i = \lambda_3} \end{cases}$$

Eigenvectors.

$$\begin{pmatrix} 2 & 0 & -2 \\ -2i & i & +2i \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

$\lambda_1 = 0$

$2x_1 - 2x_3 = 0.$ Pick $x_1 = 1 \Rightarrow x_3 = 1.$

$-2i x_1 + i x_2 - 2i x_3 = 0.$

$-2i + i x_2 + 2i = 0. \Rightarrow x_2 = 0$ $\vec{X}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$

$\cancel{i x_2 = 4i \quad x_2 = 4.}$

$\lambda_2 = 1$

$2x_1 - 2x_3 = x_1$ Pick $x_1 = 2$, $x_3 = 1.$

$x_1 = 2x_3$

$-2i x_1 + x_2 i + 2i x_3 = x_2$ $\vec{X}_2 = \begin{pmatrix} 2 \\ 1-i \\ 1 \end{pmatrix}.$

$-4i + i x_2 + 2i = x_2$

$-2i = x_2(1-i)$

$\frac{2i}{i-1} \cdot \frac{-i-1}{-i-1} = \frac{2-2i}{1+1} = 1 - i = x_2$

$\lambda_3 = i$

$2x_1 - 2x_3 = i x_1$ ~~Pick $x_3 = 1 - \frac{1}{2}i$, $x_1 = 1.$~~

$(2-i)x_1 = 2x_3.$

$-2i x_1 + i x_2 + 2i x_3 = i x_2$ $\vec{X}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$

let $x_1 = 0$, $x_3 = 0.$, $x_2 = 1$