$$\rho_0 \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda + 2\mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) - \mu\vec{\nabla}\times(\vec{\nabla}\times\vec{u}).$$

Let $\vec{u} = \vec{\nabla}\phi + \vec{\nabla}\times\psi$. Then

$$\rho_0\left[\nabla\ddot{\phi} + \vec{\nabla}\times\ddot{\vec{\psi}}\right] = (\lambda+2\mu)\vec{\nabla}\left[\nabla^2\phi + \vec{\nabla}\cdot\vec{\nabla}\times\vec{\psi}\right] - \mu\vec{\nabla}\times(\vec{\nabla}\times(\nabla\phi + \vec{\nabla}\times\psi)).$$

But $\vec{\nabla}\cdot\vec{\nabla}\times\vec{\psi} = 0$ and $\vec{\nabla}\times\vec{\nabla}\phi = 0$.

Thus

$$\rho_0\vec{\nabla}\ddot{\phi} + \rho_0\vec{\nabla}\times\ddot{\vec{\psi}} = (\lambda+2\mu)\vec{\nabla}(\nabla^2\phi) - \mu\vec{\nabla}\times\vec{\nabla}\times\vec{\psi}.$$

But $-\vec{\nabla}\times\vec{\nabla}\times\vec{\psi} = \vec{\nabla}^2\vec{\psi} - \underbrace{\vec{\nabla}(\vec{\nabla}\cdot\vec{\psi})}$.

$\vec{\nabla}\cdot\vec{\psi} = 0$ because $\vec{\psi}$ is purely rotational.

Thus

$$\rho_0\vec{\nabla}\left[\underbrace{\ddot{\phi} - (\lambda+2\mu)\nabla^2\phi}\right] + \rho_0\vec{\nabla}\times\left[\underbrace{\ddot{\vec{\psi}} - \vec{\nabla}^2\vec{\psi}}\right] = 0.$$

{compressional waves, primary, longitudinal}

{shear waves, secondary, transverse}

Evidently the speed of compressional waves is given by

$$\frac{1}{c_{comp}} = \sqrt{\frac{\rho_0}{\lambda+2\mu}} \rightarrow c_{comp} = \sqrt{\frac{\lambda+2\mu}{\rho_0}}$$

and the - - - - shear - - - - -

$$\frac{1}{c_{sh}} = \sqrt{\frac{\rho_0}{\mu}} \rightarrow c_{sh} = \sqrt{\mu/\rho_0}$$

$c_{comp} > c_{sh}$ hence

primary secondary.