want to invert:

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_1 + R_3$$

$$-1(-2)+1 = 2$$

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \div 2 \\ R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 \end{array}\right] \quad \cancel{R_3 + R_1} \quad R_2 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R_2 - 2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1/2 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R_1 - \frac{R_3}{2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & +1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R_1 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

inverse.

Invert using $A^{-1} = \frac{1}{\det A} C^T$.

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$\det A = -\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = +2$$

$$C = \begin{pmatrix} +① & -② & +③ \\ -④ & +⑤ & -⑥ \\ +⑦ & -⑧ & +⑨ \end{pmatrix} = \begin{pmatrix} -1 & 2 & 0 \\ 1 & -2 & 2 \\ 1 & 0 & 0 \end{pmatrix}$$

$① = \det \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1$

$② = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$

$③ = \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} = 0$

$④ = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$⑤ = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$

$⑥ = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$

$⑦ = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$⑧ = \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$

$⑨ = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$$C^T = \begin{pmatrix} -1 & 1 & 4 \\ 2 & -2 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

Thus $A^{-1} = \frac{1}{2} \begin{pmatrix} -1 & 1 & 1 \\ +2 & -2 & 0 \\ 0 & 2 & 0 \end{pmatrix}$

$$= \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

This matches the other approach.