Want to invert:

$$\left[\begin{array}{ccc|ccc} \boxed{\begin{matrix}-2 & 1 & 0\\ -1 & 0 & 1\\ 0 & 1 & 0\end{matrix}} & & & 1 & 0 & 0\\ & & & 0 & 1 & 0\\ & & & 0 & 0 & 1 \end{array}\right] \quad -\tfrac{1}{2}R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ -1 & 0 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_2+R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ 0 & -1/2 & 1 & -1/2 & 1 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad -2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -2 & +1 & -2 & 0\\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_3-R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -2 & 1 & -2 & 0\\ 0 & 0 & 2 & -1 & 2 & 1 \end{array}\right] \quad R_3/2$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ 0 & 1 & -2 & 1 & -2 & 0\\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right] \quad R_2+2R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/2 & 0 & -1/2 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 1\\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right] \quad R_1+\tfrac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \boxed{\begin{matrix}-1/2 & 0 & 1/2\\ 0 & 0 & 1\\ -1/2 & 1 & 1/2\end{matrix}}\\ 0 & 0 & 1 & \end{array}\right]$$

This is the inverse.

Check $\begin{pmatrix} -2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}^{-1}$ using Cramer's rule.

C is cofactor matrix.

$$A^{-1} = \frac{1}{\det(A)} C^T$$

$$\det A = -1(-2) = 2.$$

$$C^T = \begin{pmatrix} +① & -② & +③ \\ -④ & +⑤ & -⑥ \\ +⑦ & -⑧ & +⑨ \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}^T = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

$① = \det \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = -1$

$② = \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$③ = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$④ = \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$⑤ = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$⑥ = \begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} = -2$

$⑦ = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$⑧ = \begin{vmatrix} -2 & 0 \\ -1 & 1 \end{vmatrix} = -2$

$⑨ = \begin{vmatrix} -2 & 1 \\ -1 & 0 \end{vmatrix} = 1$

Thus

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1/2 & 0 & 1/2 \\ 0 & 0 & 1 \\ -1/2 & 1 & 1/2 \end{pmatrix}$$

which matches the result from row operations.