$$\boxed{\text{ODE: } y'' - 4y' + 3y = 2x e^x.}$$

→ Resonance for $r = 1$.
→ multiply form of part. sol. by x. i.e., $x(ax+b)e^x$.

$$y_p = x(ax+b)e^x = (ax^2+bx)e^x.$$

$$y' = (2ax+b)e^x + (ax^2+bx)e^x = [ax^2 + (2a+b)x + b]e^x.$$

$$y'' = (2ax + 2a + b)e^x + [ax^2 + (2a+b)x + b]e^x = [ax^2 + (4a+b)x + (2a+2b)]e^x.$$

Insert into ODE.

$$\begin{array}{l} ax^2 + (4a+b)x + (2a+2b) \\ -4ax^2 - 4(2a+b)x - 4b \\ +3ax^2 + 3bx \\ \hline = 2x. \end{array}$$

$\xrightarrow{x^2}$ $ax^2 - 4ax^2 + 3ax^2 = 0$. ↑ no information.

$\xrightarrow{x^1}$

$$4a + b - 4(2a+b) + 3b = 2$$
$$4a + b - 8a - 4b + 3b = 2$$
$$-4a = 2$$
$$a = -\tfrac{1}{2}$$

$\xrightarrow{x^0}$

$$2a + 2b - 4b = 0$$
$$2(-\tfrac{1}{2}) - 2b = 0$$
$$-1 = 2b \to b = -\tfrac{1}{2}.$$

Particular solution is therefore

$$y_p = -x\left(\tfrac{1}{2}x + \tfrac{1}{2}\right)e^x.$$

Meanwhile, ~~gen.~~ hom. solution is

$$y_h = C_1 e^x + C_2 e^{3x}.$$

Thus gen. solution is

$$y = y_p + y_h = C_1 e^x + C_2 e^{3x} - x\left(\tfrac{1}{2}x + \tfrac{1}{2}\right)e^x$$