$y_p = Ax^2 e^{-x}$

$\boxed{\text{ODE: } \quad y'' + 2y' + y = e^{-x}}$

→ Resonance twice for $r = -1$

→ multiply form of part. sol by x^2 i.e., $x^2 A e^{-x}$.

$y_p' = 2Axe^{-x} - Ax^2 e^{-x}$.

$\quad = (2Ax - Ax^2)e^{-x}$

$y_p'' = (2A - 2Ax)e^{-x} - (2Ax - Ax^2)e^{-x}$.

$\quad = (Ax^2 - 4Ax + 2A)e^{-x}$.

Put into the ODE :

$$
\begin{array}{lll}
Ax^2 & -4Ax & +2A \\
-2Ax^2 & +4Ax & \\
+Ax^2 & & \\
\hline
 & & 1
\end{array}
$$

x^2: $Ax^2 - 2Ax^2 + Ax^2 = 0$. no info.

x^1: $-4Ax + 4Ax = 0$. No info.

x^0: $2A = 1$

$A = 1/2$.

Thus $y_p = \frac{1}{2} x^2 e^{-x}$.

Meanwhile homogeneous sol. is

$y_h = C_1 e^{-x} + x C_2 e^{-x}$.

Thus the general sol. is

$y = y_p + y_h = \frac{1}{2} x^2 e^{-x} + C_1 e^{-x} + x C_2 e^{-x}$.