Rigid hollow cylindrical tube.

General solution is of the form

$$p = J_m(k_r r)\begin{Bmatrix}\cos m\theta \\ \sin m\theta\end{Bmatrix}\begin{Bmatrix}\cos k_z z \\ \sin k_z z\end{Bmatrix}\begin{Bmatrix}\cos \omega t \\ \sin \omega t\end{Bmatrix}.$$

Radial: $\partial p/\partial r\big|_{r=a} = 0 \Rightarrow J_m'(k_r a) = 0.$

$$\Rightarrow \quad k_r a = \alpha'_{mn}$$

Axial: $\partial p/\partial z\big|_{z=0} = 0 \Rightarrow$ eliminate sine term.

$\partial p/\partial z\big|_{z=L} = 0 \Rightarrow \sin k_z L = 0$

$$\rightarrow \quad k_z L = l\pi, \quad l = 0, 1, 2 \ldots$$

Thus the solution is

$$p = \sum_{l=0}^{\infty}\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} J_m(\alpha'_{mn} r/a)\, A_{mnl} \cos(m\theta + \gamma_m)$$

$$\times \cos(l\pi z/L) \times \text{time depen.}$$

Assessment of eigenfrequencies:

$$\left(\frac{2\pi f_{lmn}}{c_0}\right)^2 = k_{lmn}^2 = k_r^2 + k_z^2 = (\alpha'_{mn}/a)^2 + (l\pi/L)^2$$

$$\text{Thus} \quad f_{lmn} = \frac{c_0}{2\pi}\left[\left(\frac{\alpha'_{mn}}{a}\right)^2 + \left(\frac{l\pi}{L}\right)^2\right]^{1/2}.$$

Setting $\begin{Bmatrix} m = 0 \\ l = 0 \end{Bmatrix} \rightarrow$ radial modes ~~with harm. freq.~~

Setting $\begin{Bmatrix} m = 0 \\ n = 1 \end{Bmatrix} \rightarrow$ axial modes with harm. freq.

Not possible to have pure spinning modes.