Solve the spherical wave equation for the velocity BC.

$$u^{(r)}(r=a) = u_0 e^{j\omega t} \cos\theta .$$

which corresponds to a sphere periodically translating $\updownarrow$.

General solution for outgoing waves is the sum over

$$p_n = A_n h_n^{(2)}(kr)\, P_n(\cos\theta)\, e^{j\omega t} . \qquad (1).$$

because there is symmetry about the z-axis.

$$\rho_0 \frac{\partial u^{(r)}}{\partial t} + \frac{\partial p_r}{\partial r} = 0 \underset{\substack{\uparrow \\ \text{time-harmonic.}}}{\Longrightarrow} j\omega\rho_0 u_r = -\frac{\partial p_r}{\partial r} \Longrightarrow u_r = -\frac{1}{j\omega\rho_0}\frac{\partial p_r}{\partial r} .$$

Taking the $\partial/\partial r$ of (1) gives

$$u_r^{(r)} = -\frac{A_n k}{j\omega\rho_0} h_n^{(2)\prime}(kr)\, P_n(\cos\theta)\, e^{j\omega t}$$

$$= -\frac{A_n}{j\rho_0 c_0} h_n^{(2)\prime}(kr)\, P_n(\cos\theta)\, e^{j\omega t} . \quad \text{Therefore}$$

$$u^{(r)}(r=a) = -\frac{A_n}{j\rho_0 c_0} h_n^{(2)\prime}(ka)\, P_n(\cos\theta)\, \cancel{e^{j\omega t}} = u_0 \cancel{e^{j\omega t}} \cos\theta .$$

If one wanted one could use orth. of Legendre polys to find A_n; viz.

$$-\sum_{n=0}^{\infty} \frac{A_n}{j\rho_0 c_0} \int_0^{\pi} h_n'(ka)\, P_n(\cos\theta)\, P_m(\cos\theta) \sin\theta\, d\theta = u_0 \int_0^{\pi} \cos\theta \sin\theta \times P_m(\cos\theta)\, d\theta$$

etc.

But it can already be seen that $P_1(\cos\theta) = \cos\theta$. Thus $n=1$ is the only nonzero mode; viz.

$$-\frac{A_1}{j\rho_0 c_0} h_1'(ka)\, P_1\cancel{(\cos\theta)} = u_0 \cancel{\cos\theta}$$

$$A_1 = -\frac{u_0 j\rho_0 c_0}{h_1'(ka)}$$

Inserting into (1) gives the solution.