Uniformly pulsating cylinder: (of infinite length in z).

$u^{(r)}(r=a) = u_0 e^{j\omega t}$.

$$p_m = A_m H_m^{(2)}(k_r r) \begin{Bmatrix} \cos m\theta \\ \sin m\theta \end{Bmatrix} e^{j\omega t}.$$

There is no dependence on θ. so $m=0$.

$$p = A_0 H_0^{(2)}(k_r r)\, e^{j\omega t}.$$

To match the boundary condition take

$$u^{(r)} = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial r} = -\frac{A_0 k}{j\omega\rho_0} H_0'(ka)\cancel{e^{j\omega t}} = u_0 \cancel{e^{j\omega t}}$$

$$= \frac{A_0 k}{j\omega\rho_0} H_1(ka) = u_0$$

Thus $A_0 = \dfrac{u_0 j\rho_0 c_0}{H_1(ka)}$

Thus the solution is $p = j\rho_0 c_0 u \dfrac{H_0^{(2)}(kr)}{H_1^{(2)}(ka)} e^{j\omega t}$.

Note that $H_m^{(2)}(z) \simeq j^{m+\frac{1}{2}}\sqrt{\frac{2}{\pi z}}\, e^{-jz}$ as $z \to \infty$, to get far field expression in which $1/\sqrt{kr}$ dependence is visible. (They would have to provide asymptotes). Similarly for $ka \ll 1$ limit.

⊛ Note that impedance p/u. in near field depends on c_0 for cylindrical waves.