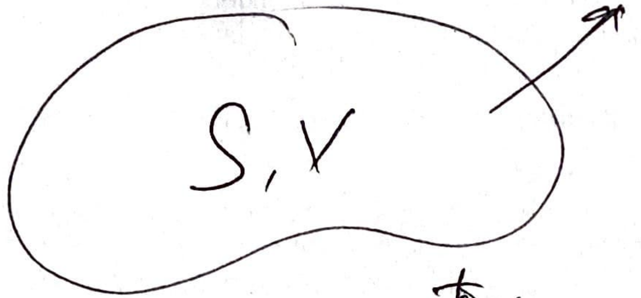
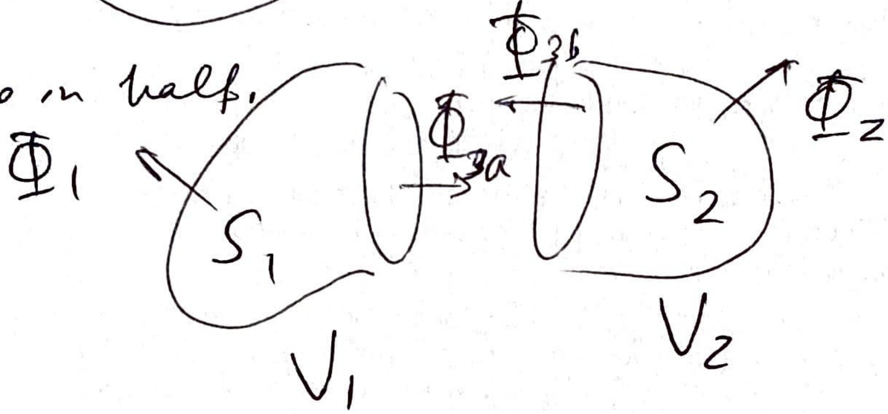
Div. thm. intuitive derivation



Flux is $\Phi(V)$.

Cut potato in half,



Total flux is now $\Phi(V_1) + \Phi(V_2) = \Phi_1 + \Phi_2 + \Phi_{3a} + \Phi_{3b}$ (1)

Note however that $\Phi_{3a} = \iint \vec{F}\cdot\hat{n}\, dS = -\iint \vec{F}\cdot(-\hat{n})\, dS = -\Phi_{3b}$.

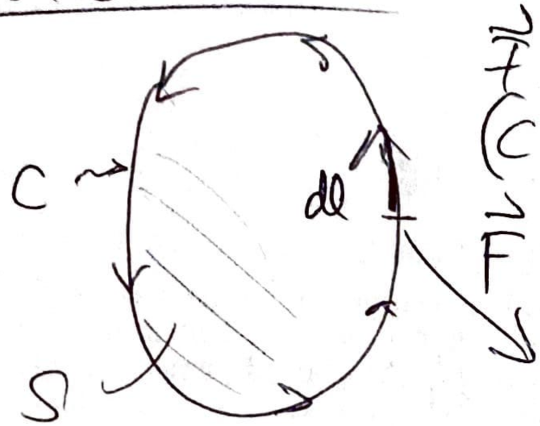
So (1) becomes $\Phi(V_1) + \Phi(V_2) = \Phi(V)$. Generalizing to more potato slices, $\Phi(V) = \sum_i \Phi(V_i) = \sum_i \iint_{S_i} \vec{F}\cdot\hat{n}\, dS$. Multiply & divide by V_i and take $i\to\infty$ limit: (note $\lim_{i\to\infty} V_i = dV$) (note $\lim_{i\to\infty} \sum_i = \int$)

$$\Phi(V) = \lim_{i\to\infty}\left[\sum_i \frac{1}{V_i} \iint_{S_i} \vec{F}\cdot\hat{n}\, dS\right] V_i$$

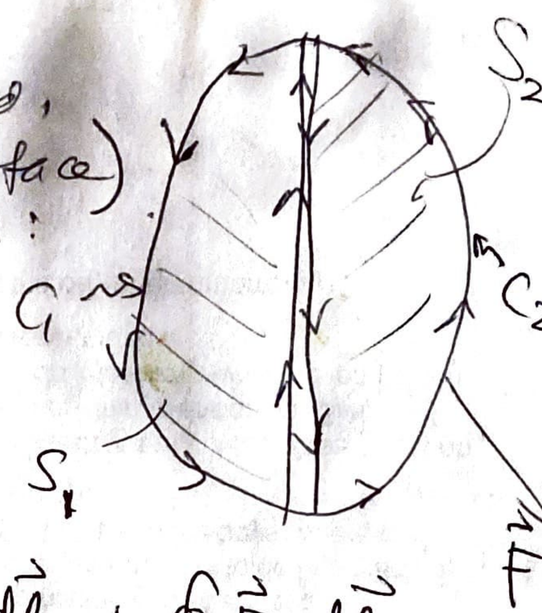
$$= \left[\frac{\iint_{S_i} \vec{F}\cdot\hat{n}\, dS}{dV}\right] dV = \iiint \vec{\nabla}\cdot\vec{F}\, dV$$

$$\iint \vec{F}\cdot\hat{n}\, dS = \iiint \vec{\nabla}\cdot\vec{F}\, dV.$$

Stokes' Theorem intuitive derivation



$\vec{F}$ is some vector field, (C is boundary, S is surface) $\Longrightarrow$ Divide into 2:



Line integral of $\vec{F}$ on C is

$$\oint_C \vec{F}\cdot d\vec{l} = \oint_{C_1} \vec{F}\cdot d\vec{l} + \oint_{C_1} \vec{F}\cdot d\vec{l}$$

Dividing into infinite number of closed loops,

$$\oint_C \vec{F}\cdot d\vec{l} = \sum_i \oint_{C_i} \vec{F}\cdot d\vec{l}$$

Recall definition of curl: $\dfrac{\oint_{C_i} \vec{F}\cdot d\vec{l}}{d\vec{S}_i} \equiv \vec{\nabla}\times\vec{F}$.

i.e., $\oint \vec{F}\cdot d\vec{l} = (\vec{\nabla}\times\vec{F})\cdot d\vec{S}_i$

$$\oint_C \vec{F}\cdot dl = \sum_i (\vec{\nabla}\times\vec{F})\cdot d\vec{S}_i$$

In limit $i\to\infty$ $\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$