Vibrating cylinder : $u^{(r)}(r=a) = \cos(\theta) u_0 e^{j\omega t}$.

general solution is

$$p_m = A_m H^{(2)}_m (kr) \begin{Bmatrix} \cos m\theta \\ \sin m\theta \end{Bmatrix} e^{j\omega t}$$

$$u^{(r)}(r=a) = -\frac{1}{j\omega\rho_0} \frac{\partial p}{\partial r} = -\frac{A_m k}{j\omega\rho_0} H_m'(ka) \begin{Bmatrix} \cos m\theta \\ \sin m\theta \end{Bmatrix} \cancel{e^{j\omega t}}$$

$$= \cos(\theta) u_0 \cancel{e^{j\omega t}}$$

By inspection, $m = 1$.

$$A_1 = -\frac{j\omega\rho_0 u_0}{k} \frac{1}{H_1'(ka)} = -j\rho_0 c_0 u_0 \frac{1}{H_1'(ka)}.$$

so $$p = -j\rho_0 c_0 u_0 \frac{H_1^{(2)}(kr)}{H_1'(ka)} \cos\theta \, e^{j\omega t}.$$

Rigorously, use orthogonality to show that $m=1$:

sine & cosine are always orthogonal; so $B_m = 0$.

$$-\frac{k}{j\omega\rho_0}\left[A_m \cos m\theta + \cancel{B_m}^{0} \sin m\theta\right] H_m'(ka) = \cos\theta \, u_0.$$

Multiply both sides by $\cos n\theta \, d\theta$ & int. from 0 to π, noting that $\int_0^\pi \cos m\theta \cdot \cos n\theta \, d\theta = \frac{\pi}{2}\delta_{nm}$

$$-H_1'(ka) \frac{A_m k}{j\omega\rho_0} = u_0 \quad \text{for } m=1,$$
$$= 0 \quad \text{for } m \neq 1.$$

Thus $A_m = -\dfrac{j\rho_0 c_0 u_0}{H_1'(ka)}$ as before.

- can take $ka \ll 1$ & $kr \gg 1$ limits to evaluate radiation of string.
- can calculate intensity of above quantity.