IntelliChoice SAT Math Camp

Algebra Basics

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1 Distributive Property

The distributive property says

$$a(b + c) = ab + ac$$

This comes up everywhere on the SAT and is often an intermediate step in algebra problems.

Apply the distributive property on the following expressions, and simplify:

1.
$$m(l + n)$$

2.
$$m(l + m)$$

Remember that $m \cdot m = m^2$. We will be talking much more about exponents!

3.
$$3bc(3c + bc^2)$$

4.
$$9x^2 + 27x^5 + 81x^7$$

Hint: Use the distributive property backwards. Factor out $9x^2$ from each term

5.

$$ab^2c^3d^4 + a^4b^3c^2d$$

Hint: Ask yourself, "What is common among these terms?"

6.

$$a^2b^4c^6d^8 + a^1b^3c^5d^7$$

7.

$$35xy^2z + 100x^3y^3z + 5xy^5z^9$$

8.

$$evol(o^{-1}v^{-1}e^{-1} + l^{-1}v^{-1}e^{-1} + l^{-1}o^{-1}e^{-1} + l^{-1}o^{-1}v^{-1})$$

Hint: Recall that $\alpha^{-1} = \frac{1}{\alpha}$

9.

$$108u^{10}v^9w^8x^7y^6z^5 + 96u^9v^8w^7x^6y^5z^4 + 84u^8v^7w^6x^5y^4z^3 + 72u^7v^6w^5x^4y^3z^2$$

10.

$$\frac{y^{-2}j^{2}}{o^{-1}}\left(j^{-1}y^{3}+\frac{j^{-3}y^{-4}}{o}\right)$$

We are going to talk lots more about how to manipulate exponents. If you are forgetting (or have not learned) the "rules" for multiplying and dividing exponents, just write out the powers of the variables above explicitly and analyze in the "traditional" way.

Use the distributive property to solve the following equations:

11.

$$5(x+4)=7(4+1)$$

12.

$$10(x+3)-100=0$$

13.

$$10x+30x^{2}=0$$

Hint: Factor out $10x$ *on the left-hand-side*
Hint: Apply the zero-product property: $0*\alpha=0$*, where* α *is any number*
Hint: There are two solutions to this equation. That is, x *can be two different numbers, both of which satisfy the equation.*

14.

$$5x^2 + 25x^3 = 0$$

Hint: Factor out $5x^2$ *on the left-hand-side.*
Hint: There are three solutions to this equation.

15. Find at least one solution to this equation:

$$x^5 - x^4 - 2x^3 = 0$$

Hint: x *can be three different numbers to satisfy this equation.*
Juniors and seniors, find all three solutions to this equation.

16.

$$(x + 1)(x - 1) + (x + 1)(2x + 1) = 0$$

Hint: There are two solutions.
Hint: Factor out $(x + 1)$ *from the entire expression.*

2 Linear equations, $y = mx + b$

The equation $y = mx + b$ is easily the most important equations in all of algebra. In fact, there is an entire branch of math called "Linear Algebra." It's all based on this simple-looking equation.

At its core, $y = mx + b$ says that y (the dependent variable) varies linearly with x (the independent variable). If x changes by, say, 4, y changes by the amount $m * 4$.

m is the slope. If m is large, then y changes a lot compared to how x changes. For example, if $m = 1000$, then if x changes by 4, y changes by $m * 4 = 1000 * 4 = 4000$.

What is b, then? b is the amount of y when $x = 0$. It is known as the y-intercept, since it the value of y at which the line touches the $y - axis$.

Graph the following lines. Label your axes, as well as the y-intercept and x-intercept.

17. $y = x$

18. $y = -2x$

19. $y = -x + 1$

20. $y = \frac{1}{2}x + 3$

Also find the equation of the line perpendicular to this line, passing through the point $(2, 4)$

21. $y = -\frac{2}{3}x - 3$

Also find the equation of the line perpendicular to this line, passing through the point $(6, -7)$

3 Systems of Linear Equations

Draw any two lines on a sheet of paper. As long as you did not draw the lines to be parallel, there will be some point at which the lines cross. This point is the solution to a system of two linear equations (each equation describing one of the lines you drew). Use any of the methods discussed in the lecture to solve these systems of equations.

22.

$$\begin{cases} y &= 2x + 3 \\ y &= -5x + 2 \end{cases}$$

23.

$$\begin{cases} 2y &= -x + 3 \\ x &= y + 2 \end{cases}$$

24.

$$\begin{cases} 500y &= -5000x + 50 \\ y &= -10x + \frac{1}{10} \end{cases}$$