IntelliChoice SAT Math Camp

Inequalities, Absolute Value, Dimensional Analysis

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1 Inequalities

Equations are very specific statements. Solutions to equations are discrete. For example, problem 15 had three distinct values of x that satisfied the equation. Inequalities, on the other hand, are more general statements. Solutions to inequalities are continuous. For example, the solution to the inequality $x + 1 > 1$ are all real numbers that are greater than 0. There is an infinite number of solutions!

Inequalities and equations are treated very similarly. But we have to be careful about our (+) and (-) signs: the equals sign doesn't have a sense of direction, but the inequality does. Consider the following statement:

$$a = b$$

Multiplying by -1 on both sides,

$$-a = -b$$

Contrast this with the statement:

$$a > b$$

Multiplying by -1 on both sides,

$$-a < -b$$

Notice the "greater than" sign has become a "less than" sign. If you are doubtful of this result, try plugging in two of your favorite numbers into $a > b$ (for example, $a = 3$ and $b = 2$). Check for yourself that $-a < -b$ ($-3 < -2$). The main result is that we must *change the direction of the inequality* if we multiply by -1 across the inequality.

Solve and graph (in 1D–along a number line) the following inequalities:
1.

$$2x + 3 > 4$$

2.

$$-10x + 4 \leqslant 100$$

3.

$$10x \geqslant -100$$

Graph the following inequalities in 2D: 3.

$$y \leqslant 10x - 5$$

4.

$$x > 2y + 7$$

5.

$$0 \leqslant 5x - 15y$$

2 Absolute Value

The absolute value of a number c is denoted by vertical bars: $|c|$. There are several definitions of absolute value. They are equivalent, but depending on the context, some are more convenient than others. The most intuitive definition is the following:

The absolute value of a number is its distance from 0.

In other words, the absolute value of a number cannot be less than zero. The absolute value of -5, therefore is $|-5| = 5$.

6.

$$|-3| =$$

7.

$$|\pi| =$$

8.

$$|-e| =$$

What if we put a function inside a pair of absolute value bars? Say we had $|x+1|$. Does it make sense for an entire function to have a "distance from the origin"? No: a function represents an infinite number of values, each of which has its own distance from the origin. How would we evaluate $|x+1|$? We need a more precise definition:

$$|c| \equiv \begin{cases} c & \text{if } c \geqslant 0 \\ -c & \text{if } c < 0 \end{cases}$$

For the case of $|x+1|$,

$$|x+1| \equiv \begin{cases} x+1 & \text{if } x+1 \geqslant 0 \\ \implies x-(x+1) & \text{if } x+1 < 0 \end{cases}$$

This takes some time to get your mind around. Plug in some values for x and see if the above definition holds.

9.

$$|x+1| = 3$$

10.

$$|2x-10| = 20$$

11.

$$5|x| = 3 + x$$

12.

$$x + |x + 1| = 0$$

How do we makes sense of this last result?

Draw 1D graphs for the following: 13.

$$1 + |2x + 1| > -4$$

How do we makes sense of this last result?

14.

$$|x - 1| \leqslant 0$$

Hint: Be careful about $x = 1$!

15.

$$|x| > 0$$

3 Dimensional Analysis.

In high school (and in life), you will be confronted with many ways to measure the same physical quantities.

We like to express physical quantities in units that require writing the least number of digits. For example, would you rather express the distance from Earth to the Sun in meters ($149,600,000,000$ meters) or light-minutes (8.5 light-minutes)? It's much easier to write the latter.
Circle the most appropriate unit of measure for the following quantities:

16. Amount of time in a day:

  a. years

b. gigaseconds ($*10^9$ seconds)

c. microhours ($*10^{-6}$ hrs)

d. hours

e. months

17. Amount of water contained in a bathtub

a. cups

b. gallons

c. quarts

d. pints

e. teaspoons

18. Speed of an interstellar spaceship

a. inches/second

b. meters/second

c. decameters/second (1 Dm = 10 m)

d. angstroms/second (1 Å = 10^{-10} m)

e. kilometers/second

19. Distance from your home to school

a. meters

b. gigameters

c. megameters

d. decimeters

e. kilometers

If we want to write a physical quantity using the best unit, then we need to be able to convert from one unit to another. *This is done by multiplying by a fancy form of the number "1."*

Fact 1. $\frac{\beta}{\beta} = 1, \beta \neq 0$. That is, any number (except 0) divided by itself is 1.

Fact 2. $\alpha * 1 = \alpha$. That is, multiplying any number by "1" does nothing to the value of any number.

Let's use these two facts to convert from 235 centimeters to meters:

$$235\,\mathrm{cm} * \frac{1\,\mathrm{m}}{100\,\mathrm{cm}} = 2.35\,\mathrm{m}$$

We used **fact 1** to write the number "1" as $\frac{1\,\mathrm{m}}{100\,\mathrm{cm}}$; we used **fact 2** to to multiply 235 cm by 1.

Convert the following quantities to the requested units. Use a calculator for the computations and round numbers to the nearest *tenth* (0.1).

20. 31415 meters = ? kilometers

21. 0.125 feet = ? inches

22. 500 grams = ? pounds
(Remember that 1 kilogram = 2.2 lbs and that 1000 grams = 1 kg)

23. 100 inches = ? decameters
(Remember that 2.54 cm = 1 in. and that 10 m = 1 Dm)

24. 0.0000000001 lightyear = ? megameters
(1 parsec = 3.26 lightyears, 1 parsec = $3.086 * 10^{16}$ meters, and 10^6 meters = 1 megameter)