IntelliChoice SAT Math Camp

Quadratics

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1 Definition

So far, we have solved linear equations. These are equations whose highest power (what mathematicians call the "degree") is 1. These equations were of the form

$$y = Ax^1 + Bx^0 \tag{1}$$

Notice there are two coefficients that define linear equations: A and B. Also note that what I've written is just $y = Ax + B$, but I've included the powers 1 and 0 for a reason you will see soon.

We will now venture into quadratic equations. These are equations whose degree is 2. They are of the form

$$y = Ax^2 + Bx^1 + Cx^0 \tag{2}$$

Notice that there are three coefficients that define quadratic equations: A, B, and C.

1. How does the form of (2) compare to (1)?

2 Solution

Just as the solution to a linear equation $y = Ax + B$ is

$$x = \frac{y - B}{A}$$

the solution to a quadratic equation $y = Ax^2 + Bx + C$ is

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{3}$$

$\sqrt{B^2 - 4AC}$ is called the *discriminant*.

2. How many real distinct solutions are there when the discriminant is positive?

3. How many real distinct solutions are there when the discriminant is 0?

4. How many real distinct solutions are there when the discriminant is negative?

Solve the following quadratic equations using any method. Using (3), while formulaic, is often not the fastest way of solving these problems! With some practice, you will be able to guess these solutions to almost all the quadratic equations tested on the SAT.

Once you have solved the equation, graph the behavior of y as a function of x.

5.

$$y = x^2 - 1$$

6.

$$y = x^2 + 2x + 1$$

7.

$$y = x^2 - 2x + 1$$

8.

$$y = x^2 - 4$$

9.

$$y = x^2 + 2x - 3$$

10.

$$y = x^2 - 15x + 50$$

11.

$$y = 2x^2 - x - 3$$

12.

$$y = 5x^2 - 36x + 7$$

13.

$$y = -3x^2 + 18x - 27$$

14.

$$y = 100x^2 - 10001x + 100$$

15.

$$y = 50x^2 + 75x + 25$$

16.

$$y = 3x^2 + 2x - 1$$

17.

$$y = 10x^2 - 9x - 1$$

18.

$$y = 6x^2 - 5x - 1$$