IntelliChoice SAT Math Camp
Exponents & Logarithms
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Exponents. There are only three properties to know about exponents:

Property 1. $\alpha^{-\beta} = \frac{1}{\alpha^{\beta}}$.

Property 2. $\alpha^{\beta}\alpha^{\gamma} = \alpha^{\beta+\gamma}$.

Property 3. $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$.

Use these three properties to simplify the following expressions:

1. $\pi x^{-\pi}$

2. $((blues)^8)^{-1/16}$

3. $n^b n^{-b} b^n b^{-n}$

4. $((2x)^{18})^{-9}$

5. $(10x^2)^5(32x^4)^{-4}$

6. $\sqrt{\dfrac{h^{11}o^{22}p^{33}e^{44}}{h^9 o^{20}p^{31}e^{42}}}$

Roots. The nth root of x is written as $\sqrt[n]{x}$ (or $x^{\frac{1}{n}}$–the two notations are equivalent). $\sqrt[n]{x}$ is the number that, when multiplied by itself n times, equals x. For example,

When $n = 2$, we retrieve the *square root*. Since this is the most common root, we write $\sqrt{x}$ instead of $\sqrt[2]{x}$. For example, $\sqrt[4]{81} = 3$, since 3 is the the number that, when multiplied by itself 4 times, equals 81.

Simplify these expressions:

7. $\sqrt{100x^{1000}y^{100}}$

8. $\sqrt{1000x^3y^4}$

9. $\sqrt[4]{243a^4b^6}$

10. $(l^2o^4v^6e^8)^{\frac{1}{2}}$

11. $(joy)^{\frac{2}{3}}\sqrt[2]{(joy)}^3$

Write using exponents:

12. $\sqrt{\sqrt{a}}$

13. $\sqrt[10]{\sqrt[5]{a^{100}}}$

1   Logarithms

The logarithm of a number is an exponent. Specifically, $\log_b a = c$ asks the question, "What number b, when raised to the power of c equals a?"

For example,
$\log_{10} 1 = 0$ because $10^0 = 1$.
$\log_{10} 100 = 2$ because $10^2 = 100$.

b is called the base. In most cases on the SAT, $b = 10$.

Just as there are three properties to know for exponents, there are three properties to know for logarithms:

Property 1. $\log_b(a^c) = c \log_b a$.

Property 2. $\log_b a + \log_b c = \log_b ac$.

Property 3. If $\log_b a = c$, then $c = \frac{\log_\beta a}{\log_\beta b}$ where β is a base of your choosing.

Compute the following logarithms:

14. $\log_{10} 1000 =$

15. $\log_2 8 =$

16. $\log_5 625 =$

17. $\log_{10} 10 =$

18. $\log_{10} 1 =$
Hint: Recall that $\alpha^0 = 1$

19. $\log_a a =$

20. $\log_\pi \pi^2 =$

Compute the following logarithms:

21. $\log_{10}(10^{10})$

22. $\log_\pi(\pi^{2\pi}) + \log_\pi(\pi^{-2\pi})$

23. $5\log_5 25 + 5\log_5 \frac{1}{25}$

24. $\log_{100} 10 + \log_{1000} 100$
Hint: Use **Property 3** *on the right-side logarithm.*

25. $\log\left(\sqrt{10000}\right)^2$
Hint: You should assume the base is 10 if not specified.

26. $\left(\log\sqrt{10000}\right)^2$
Hint: This is not the same as question 25! Look carefully