WAVE PHENOMENA

ME/EE 384N-8—Spring 2024

Comments on the Fourier Acoustic Propagation Code

The posted Matlab code FourierAcousticProp.m uses the angular spectrum method to compute the acoustic field radiated by a planar velocity source. All quantities in the code are made dimensionless through normalization by a characteristic peak source velocity u_0, source dimension a, and diffraction length (Rayleigh distance) $z_0 = \frac{1}{2}ka^2$. The dimensionless coordinates are $X = x/a$, $Y = y/a$, and $Z = z/z_0$; the wave numbers are $K = ka$ and $K_i = k_i a$, where i represents the x, y, or z component; and the field variables are $P = p/\rho_0 c_0 u_0$ and $U_i = u_i/u_0$. The dimensionless source velocity in the plane $Z = 0$ is designated here by $U_0(X,Y)$, which is taken to have a peak amplitude of unity, and which tends to zero for $X^2 + Y^2$ somewhat greater than unity. With the quantities normalized in this way, for $ka \gg 1$, and assuming no focusing or beam steering, the acoustic field should have a magnitude of order one in the region defined by $-1 \lesssim X \lesssim 1$, $-1 \lesssim Y \lesssim 1$, and $0 \le Z \lesssim 1$, and it should fall off outside this region. The acoustic pressure and particle velocity components in any plane $Z > 0$ are computed as follows:

$$P(X,Y,Z) = \mathcal{F}^{-1}\left\{\mathcal{F}\left[U_0(X,Y)\right]\frac{K}{K_z}e^{iK_zKZ/2}\right\}$$
$$U_i(X,Y,Z) = \mathcal{F}^{-1}\left\{\mathcal{F}\left[U_0(X,Y)\right]\frac{K_i}{K_z}e^{iK_zKZ/2}\right\}$$

where $K_z = (K^2 - K_x^2 - K_y^2)^{1/2}$. Normalized time-averaged intensity components $J_i = I_i/\rho_0 c_0 u_0^2$ may be computed using the relation $J_i = \frac{1}{2}\,\mathrm{Re}\,PU_i^*$. All of the dimensionless field variables are thus uniquely determined by $U_0(X,Y)$ and ka.

To acquire a feel for the computations, use the “super-Gaussian” source function $U_0(R) = \exp(-R^n)$ with $n = 30$, where $R = (X^2+Y^2)^{1/2}$, let $ka = 50$, define the field size by $|X|, |Y| \le 20$, use 512×512 points in the Fast Fourier Transform (FFT), and compute the sound pressure in the X-Y plane at distance $Z_{xy} = 0.3$, which is approximately where the last maximum occurs along the axis of a circular piston. Any departures from circular symmetry that you observe (apart from the rectangular discretization pattern) in either the angular spectrum or the pressure field are due to the finite width of the angular spectrum being calculated, and the finite region of the source plane being sampled. Use of the FFT causes both the angular spectrum and the acoustic field variables to become periodic functions that are repeated like squares on a checkerboard. If the magnitudes of the angular spectrum and acoustic field are not sufficiently small at the edges of their respective regions, then overlap from neighboring regions tends to manifest itself as ripples. For example, as you increase the exponent n in the super-Gaussian source function (which approaches the step function for a circular piston as $n \to \infty$) you should observe increased rippling in both k-space and physical space. This is caused by broadening of the angular spectrum associated with sharpening of the edge of the source velocity distribution. Likewise, reducing ka increases rippling in the acoustic field as the beam becomes broader. With these factors in mind, you can explore the effects of changing the field width and spatial sampling interval on the computed pressure field at various distances from the source.