Sound in a general axisymmetric ($m = 0$) spherical enclosure

Let the radius of the spherical source be a. At the boundary of the sphere, $r = a$, there is a pressure source given by

$$p(a, \theta) = p_0 f(\theta) e^{j\omega t} \tag{1}$$

Equation (1) can be written as an expansion of Legendre polynomials, where M_n is the expansion coefficient:

$$p(a, \theta, t) = p_0 \sum_{n=0}^{\infty} M_n P_n(\cos\theta) e^{j\omega t} \tag{2}$$

Meanwhile, the pressure inside the sphere is written in terms of Legendre polynomials and spherical Bessel functions:

$$p(r, \theta, t) = \sum_{n=0}^{\infty} A_n P_n(\cos\theta) j_n(kr) e^{j\omega t} \tag{3}$$

Matching equations (2) and (3) at $r = a$, and canceling the time-dependence,

$$p_0 \sum_{n=0}^{\infty} M_n P_n(\cos\theta) = \sum_{n=0}^{\infty} A_n P_n(\cos\theta) j_n(ka) \tag{4}$$

where, by the orthogonality of Legendre polynomials,

$$M_n = \frac{2n+1}{2} \int_0^{\infty} f(\theta) P_n(\cos\theta) \sin\theta \, \mathrm{d}\theta. \tag{5}$$

Equation (4) is solved for A_n:

$$A_n = \frac{p_0 M_n}{j_n(ka)} \tag{6}$$

Substituting equation (6) into equation (3) results in the solution

$$p(r, \theta, t) = p_0 \sum_{n=0}^{\infty} M_n P_n(\cos\theta) \frac{j_n(kr)}{j_n(ka)} e^{j\omega t} \tag{7}$$

where M_n is given by equation (5).

Comment from MFH: "This is like the 1-D problem in Acoustics I for a source at the end of a closed tube. So like the tube problem, your resonances should be the frequencies for which $j_n(ka) = 0$ in the denominator of Eq. (7)....And if you used a velocity source, it should be $j_n'(ka) = 0$."