Radiation Force

History, Theory, and Recent Advances

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Outline

History

Waves on a string

Acoustic waves

Conclusion

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Early observations of comets

- Ancient and Medieval Chinese astronomers carefully observed comets.
- Comets were recorded for their astrological significance.
- They noted that comet tails point away from the Sun in 66 AD,[1] when the appearance of Halley's Comet was first accurately recorded.[2]

Record of seven comets from 2nd century B.C. (Han Dynasty)

[1] F. Mignard. *Interrelations Between Physics and Dynamics for Minor Bodies in the Solar System*. Gif-sur-Yvette: Editions Frontières, 1992, pp. 419–451.

[2] J. Needham and C. A. Ronan. Vol. 3. Cambridge University Press, 1978.

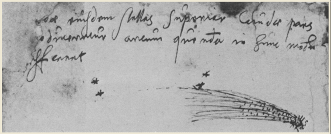
The Great Comet of 1577: Tycho Brahe

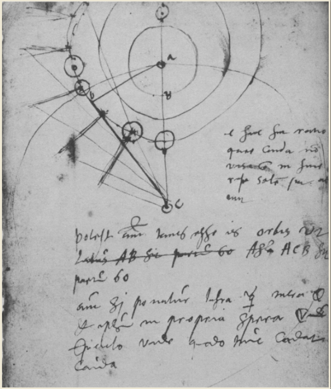
- Brahe measured the comet's parallax and observed the orientation of its tail.

- "It was located far beyond those regions and even beyond the moon, in the celestial regions which Aristotle had described as the realm of eternal, unchanging circular motion."[a]

  All have turned their tails away from the sun... the tail of a comet is nothing but the rays of the sun which have passed through the body of the comet... it holds a part of the radiance from the sun within itself.

[a] J. R. Christianson. *Isis* 70 (1979), pp. 110–140.





J. R. Christianson. *Isis* 70 (1979), pp. 110–140

The Great Comet of 1577: Johannes Kepler

- Katharina Kepler (1547–1622) introduced her six-year-old son Johannes to comets in 1577.

  I was taken by my mother to a high place to look at it.[3]

J. W. Fehrle. `https://en.wikipedia.org/wiki/Katharina_Kepler`.

Tragopogon. `https://en.wikipedia.org/wiki/Katharina_Kepler`.

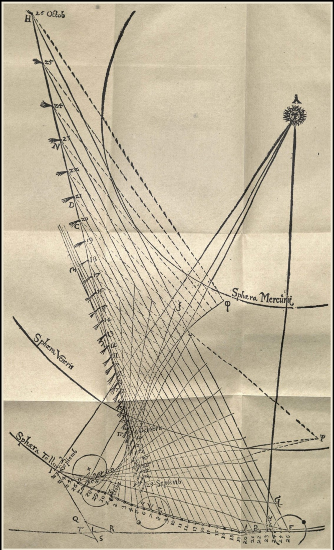
[3] A. Koestler. London: Hutchinson, 1959.

Kepler and the Comets of 1607 (Halley) and 1618

- Kepler provided a more accurate physical explanation of comet tails in 1619[a]

  The matter of a comet is gradually destroyed by the sunlight and driven away in the direction of the solar rays to form the tail.[b]

  Given ships or sails adapted to the heavenly breezes, there will be some who will not fear even that void... So, for those who will come shortly to attempt this journey, let us establish the astronomy: Galileo, you of Jupiter, I of the moon.[c]



J. Kepler. A Apergeri, AvgvstæVindelicorvum, 1619

[a] J. Kepler. A Apergeri, AvgvstæVindelicorvum, 1619.

[b] F. Mignard. *Interrelations Between Physics and Dynamics for Minor Bodies in the Solar System.* Gif-sur-Yvette: Editions Frontières, 1992, pp. 419–451.

[c] J. Kepler. Apud D. Zachariam Palthenium, 1611. URL: https://archive.org/details/ioanniskeplerima00kepl/mode/2up.

Leonhard Euler's explanation

Sound vigorously excites not only a vibratory motion in the air particles... but also a real motion in small, very light dust particles which tumble in the air. It cannot be doubted that the vibratory motion caused by the light produces a similar effect.[4]

Leonhard Euler

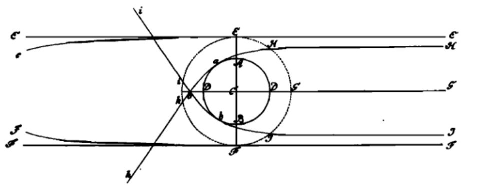


FIG. 1. Displacement of the outer portions of comet by radiation force as envisaged by Euler in 1746.[3] ADBD = solid core of comet; ESF = limit of atmosphere of comet; EHGJF are sun's rays, entering from the right.

R. T. Beyer. *J. Acoust. Soc. Am.* 63 (1978), pp. 1025–1030

[4] L. Euler. *Mem. Acad. R. Sci. Classe Math. Berlin* 117 (1746).

Radiation force in the 19th century

- Particles in standing waves accumulate towards the nodes.[5]

  In a medium in which waves are propagated, there is a pressure in the direction normal to the waves, and numerically equal to the energy in unit of volume.[6]

- Electromagnetic radiation pressure was experimentally demonstrated (the first experimental validation of Maxwell's theory).[7]

August Kundt

James Clerk Maxwell

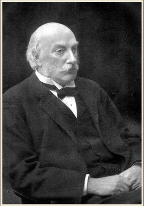
Pyotr Nikolaevich Lebedev

[5] A. Kundt. *Ann. Phys.* 203 (1866), pp. 497–523.
[6] J. C. Maxwell. Oxford: Clarendon Press, 1873, Sec. 792.
[7] P. Lebedev. *Ann. Phys.* 6 (1901), pp. 433–458.

Radiation force in the 20th century

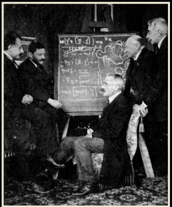
That æthereal vibrations must exercise a pressure upon a perfectly conducting…boundary was Maxwell's deduction from his general equations of the electromagnetic field…It would be of interest to inquire whether other kinds of vibration exercise a pressure, and if possible to frame a general theory of the action."[8]

- Pendulums and strings were used to simplify the analysis.[9]
- Langevin calculated acoustic radiation forces exerted in free space.
- Beth measured the angular momentum of light.[10]



Lord Rayleigh

Léon Brillouin



Paul Langevin

[8] Lord Rayleigh. *Philos. Mag.* 3 (1902), pp. 338–346.

[9] Lord Rayleigh. *Philos. Mag.* 3 (1905), pp. 364–374; L. Brillouin. *Ann. Phys.* 4 (1925), pp. 528–586.

[10] R. A. Beth. *Phys. Rev.* 50 (1936), pp. 115–125.

Radiation force in space

- Taylor Wang flew on the *Challenger* (STS-51-B) in 1985 to perform microgravity experiments involving radiation force due to standing acoustic waves to measure material properties.[11]
- Subsequent radiation force experiments were conducted in 1992 by Eugene Trinh aboard the *Columbia* (STS-50) to study acoustic manipulation of drops and their surface properties.
- “Readers should note the significance of his initials: ‘E. T.’ ”[12]

Challenger launch (STS-51-B)

Taylor Wang aboard the *Challenger*

Eugene Trinh aboard the *Columbia*

[11] T. G. Wang and C. P. Lee. *Nonlinear Acoustics*. Ed. by M. F. Hamilton and D. T. Blackstock. 3rd. Cham, Switzerland: Springer, 2024. Chap. 6.

[12] Acoustical Society of America. *Echoes* 2 (1992), pp. 1–8.

Outline

But comets are vague creatures.[13]

History

Waves on a string

Acoustic waves

Conclusion

[13] J. H. Poynting. *Philos. Mag.* 9 (1905), pp. 393–406.

Wave equation

- For small transverse displacements ξ, Newton's second law reduces to

$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{c_0^2}\frac{\partial^2 \xi}{\partial t^2} = 0, \qquad c_0 = \sqrt{\mathcal{T}/\rho_0}. \tag{1}$$

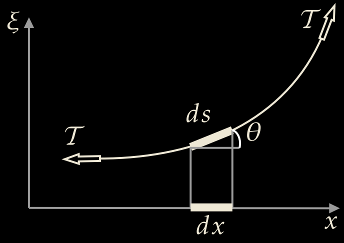
- The kinetic energy density is (by definition)

$$T = \tfrac{1}{2}\rho_0(\partial\xi/\partial t)^2. \tag{2}$$

- The potential energy density is (from the definition of work)

$$U = \tfrac{1}{2}\mathcal{T}(\partial\xi/\partial x)^2. \tag{3}$$

- The Lagrangian density is $L = T - U$.



Momentum conservation

- Multiplying Eq. (1) by $\partial\xi/\partial x$ yields

$$\boxed{\frac{\partial g}{\partial t} = \frac{\partial S}{\partial x},} \tag{4}$$

where the *radiation stress* is

$$S \equiv L - \rho_0(\partial\xi/\partial t)^2, \tag{5}$$

and where the *momentum density* is

$$g \equiv I/c_0^2, \quad [I = -\mathcal{T}(\partial\xi/\partial t)(\partial\xi/\partial x) = \text{ intensity}] \tag{6}$$

- The integral form of Eq. (4) is

$$S(x_2) - S(x_1) - \frac{d}{dt}\int_{x_1}^{x_2} g\,dx = 0. \tag{7}$$

- Equation (7) states that no momentum is lost as a wave passes unperturbed from points x_1 to x_2.

Radiation force

- Equation (7) holds in free space (Newton's 3rd law).
- If object sits between x_1 and x_2, then Eq. (4) is force density.
- For time-harmonic solutions ξ, $\langle \frac{d}{dt} \int_{x_1}^{x_2} g\, dx \rangle = 0$, so

$$\boxed{F = \langle S(x_2) - S(x_1) \rangle.} \tag{8}$$

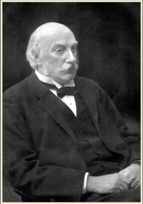
[F]or the calculation of the average force correct up to terms of the second order in the velocity, it is sufficient to find the solution of the linear scattering problem.[14]

G. Boebinger, S. Iordansky, D. Pines, and L. Pitaevskii. *Physics Today* 70 (2017), pp. 68–69

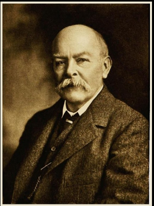
[14] L. P. Gor'kov. *Sov. Phys. Dokl.* 6 (1962), pp. 773–775.

. . . or not

Lord Rayleigh kindly pulled me out of the pit into which I fell, pointing out that when we take into account second-order quantities the ordinary sound equation does not hold.[15]



Lord Rayleigh



John Henry Poynting

What is the equation of motion for a string at quadratic order?

Answer: $0 = 0$. The leading-order nonlinearity is cubic:

$$\frac{\partial^2 \xi}{\partial x^2}\left[1 - \frac{3}{2}\left(\frac{\partial \xi}{\partial x}\right)^2\right] - \frac{1}{c_0^2}\frac{\partial^2 \xi}{\partial t^2} = 0\,. \tag{9}$$

[15] J. H. Poynting. *Philos. Mag.* 9 (1905), pp. 393–406.

Comments on Rayleigh radiation force

For the plane wave without transverse gradients of intensity, the Rayleigh formula is valid, whereas for a narrow acoustic beam in which an average acoustic pressure across the beam is different from the external pressure, the Langevin formula works.[16]

The radiation pressure is Langevin if it depends on the sound waves, and it is Rayleigh if it depends on the sound waves plus a constraint.[17]

The Rayleigh radiation force has no major practical importance. It is mentioned here only because it is invariably discussed alongside the Langevin radiation force in the theoretical literature.[18]

The results that follow are all *Langevin* radiation forces.

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[16] L. A. Ostrovsky. *Radiophysics and Quantum Electronics* 66 (2023), pp. 421–430.
[17] T. G. Wang and C. P. Lee. *Nonlinear Acoustics*. Ed. by M. F. Hamilton and D. T. Blackstock. 3rd. Cham, Switzerland: Springer, 2024. Chap. 6.
[18] G. R. Torr. *Am. J. Phys.* 52 (1984), pp. 402–408.

Progressive wave radiation force on point mass

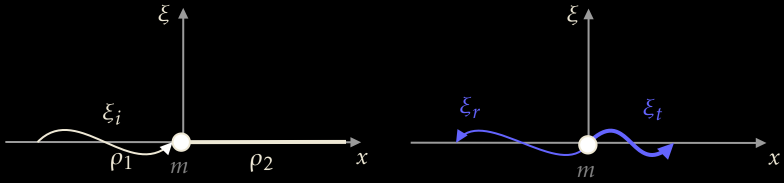
- Assume $\xi = \Re e(\tilde{\xi} e^{-i\omega t})$. The boundary conditions at $x = 0$ are[19]

$$\xi_i + \xi_r = \xi_t , \tag{10}$$

$$m \frac{\partial^2 \xi_t}{\partial t^2} = \mathcal{T} \left[\frac{\partial \xi_t}{\partial x} - \frac{\partial(\xi_i + \xi_r)}{\partial x} \right]. \tag{11}$$

- Denote $\tilde{\xi}_\mathrm{i} = A_\mathrm{i} e^{ik_1 x}$, $\tilde{\xi}_\mathrm{r} = A_\mathrm{r} e^{-ik_1 x}$, and $\tilde{\xi}_\mathrm{t} = A_\mathrm{t} e^{ik_2 x}$:

$$\xi_\omega = \begin{cases} A_\mathrm{i} e^{ik_1 x} + A_\mathrm{r} e^{-ik_1 x} , & x < 0 , \\ A_\mathrm{t} e^{ik_2 x} , & x > 0 . \end{cases} \tag{12}$$



[19] D. J. Griffiths. 3rd. Upper Saddle River, New Jersey: Pearson, 1999, Prob. 9.6(a).

Progressive wave radiation force on point mass

- Combining Eqs. (10), (11), and (12) yields ref.-trans. coefficients

$$\mathscr{R} = \frac{A_\mathrm{r}}{A_\mathrm{i}} = \frac{1-(k_2/k_1)+imk_1/\rho_1}{1+(k_2/k_1)-imk_1/\rho_1}\,, \tag{13}$$

$$\mathscr{T} = \frac{A_\mathrm{t}}{A_\mathrm{i}} = \frac{2}{1+(k_2/k_1)-imk_1/\rho_1}\,. \tag{14}$$

- Evaluating Eq. (8) in terms of Eq. (12) yields the radiation force

$$\boxed{\begin{aligned} F &= \tfrac{1}{2}\mathcal{T}\xi_0^2\Big[k_1^2(1+|\mathscr{R}|^2)-k_2^2|\mathscr{T}|^2\Big] \\ &\to 2\langle E_\mathrm{i}\rangle, \quad m\to\infty \text{ (or } k_2\gg k_1), \\ &\to 2\langle E_\mathrm{i}\rangle, \quad m\to 0 \text{ and } k_1\gg k_2\,, \\ &\to \langle E_\mathrm{i}\rangle, \quad \mathscr{R}=\mathscr{T}=0 \text{ (controversial)}, \\ &\to 0\,, \quad m\to 0 \text{ and } k_1=k_2\,, \end{aligned}} \tag{15}$$

where $\mathscr{R}$ and $\mathscr{T}$ are given by Eqs. (13) and (14) and $\langle E_\mathrm{i}\rangle = \frac{1}{2}\xi_0^2\mathcal{T}k_1^2$.[20]

[20] T. G. Wang and C. P. Lee. *Nonlinear Acoustics*. Ed. by M. F. Hamilton and D. T. Blackstock. 3rd. Cham, Switzerland: Springer, 2024. Chap. 6.

Physical interpretation of radiation force

- For a perfect absorber, it was found that $\langle F \rangle = \langle E_\mathrm{i} \rangle$.
- The same result can be obtained qualitatively by recalling that

$$\langle I \rangle = \tfrac{1}{2}\mathcal{T} k_1^2 c_1 \xi_0^2 = \frac{\text{work}}{\text{time}}. \tag{16}$$

- In 1D, intensity equals power, the rate at which work is done:

$$\langle I \rangle = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}} = \text{force} \times \text{speed} = \langle F \rangle c_1, \tag{17}$$

- Combining Eqs. (16) and (17) yields

$$\langle F \rangle = \frac{\langle I \rangle}{c_1} = \tfrac{1}{2}\mathcal{T} k_1^2 \xi_0^2 = \langle E_\mathrm{i} \rangle.$$

- "On a perfect reflector the pressure is twice as great, because the momentum switches direction, instead of simply being absorbed."[21]

[21] D. J. Griffiths. 3rd. Upper Saddle River, New Jersey: Pearson, 1999, p. 382.

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Maxell commercial (1981)

YouTube clip

Back to the Future (1985)

YouTube clip

Quadratic quantities in acoustics

TEXAS The University of Texas at Austin

- For waves on a string, there is only one field variable, ξ.
- In acoustics, there are two field variables, p and $\mathbf{v}$.
- For 1D problems, $S_{xx} = -\langle P^{\mathrm{L}} - P_0 \rangle = -\langle E \rangle$.[22]

Quantity	Waves on a string	Acoustic waves
kinetic energy	$T = \frac{1}{2}\rho_0(\partial\xi/\partial t)^2$	$T = \frac{1}{2}\rho_0 v^2$
potential energy	$U = \frac{1}{2}\mathcal{T}(\partial\xi/\partial x)^2$	$U = \frac{1}{2}p^2/\rho_0 c_0^2$
intensity	$I = -\mathcal{T}(\partial\xi/\partial t)(\partial\xi/\partial x)$	$\mathbf{I} = p\mathbf{v}$
momentum density	$g = I/c_0^2$	$\mathbf{g} = \mathbf{I}/c_0^2$
radiation stress	$S = L - \rho_0(\partial\xi/\partial t)^2$	$\underline{\mathbf{S}} = \underline{\mathbf{I}}L - \rho_0\mathbf{v}\otimes\mathbf{v}$
energy conservation	$\partial I/\partial x + \partial E/\partial t = 0$	$\nabla\cdot\mathbf{I} + \partial E/\partial t = 0$
momentum conservation	$\partial g/\partial t = \partial S/\partial x$	$\partial\mathbf{g}/\partial t = \nabla\cdot\underline{\mathbf{S}}$
radiation force	$F = \langle S(b) - S(a)\rangle$	$\mathbf{F} = \oint\langle\underline{\mathbf{S}}\rangle\cdot d\mathbf{A}$

[22] T. G. Wang and C. P. Lee. *Nonlinear Acoustics*. Ed. by M. F. Hamilton and D. T. Blackstock. 3rd. Cham, Switzerland: Springer, 2024. Chap. 6.

Progressive wave force at fluid interface

- Let $p = \Re\mathfrak{e}(\tilde{p}e^{-i\omega t})$ and $v = \Re\mathfrak{e}(\tilde{v}e^{-i\omega t})$, and assume the form

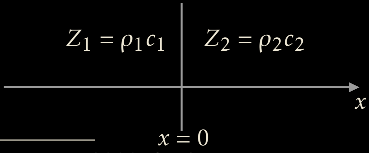
$$\tilde{p}_\mathrm{i} = p_0 e^{ik_1 x}, \quad \tilde{p}_\mathrm{r} = p_0 \mathscr{R} e^{-ik_1 x}, \quad \tilde{p}_\mathrm{t} = p_0 \mathscr{T} e^{ik_2 x}. \tag{18}$$

- The reflection and transmission coefficients are[23]

$$\mathscr{R} = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \quad \mathscr{T} = \frac{2Z_2}{Z_1 + Z_2}. \tag{19}$$

- The radiation pressure *difference* equals $\langle S(x>0) - S(x<0)\rangle$:

$$\boxed{\begin{aligned} \langle \Delta P \rangle &= \frac{p_0^2}{2}\left(\frac{1+\mathscr{R}^2}{\rho_1 c_1^2} - \frac{\mathscr{T}^2}{\rho_2 c_2^2}\right) \\ &\to 2\langle E_\mathrm{i}\rangle, \quad Z_2 \gg Z_1 \text{ or } Z_2 \ll Z_1 \\ &\to \langle E_\mathrm{i}\rangle, \quad \mathscr{R} = \mathscr{T} = 0 \text{ (controversial)} \end{aligned}} \tag{20}$$



[23] D. T. Blackstock. New York: Wiley, 2000, Ch. 3.

Demo: Sound beam force at fluid interface

- For incident plane wave on water-air interface with cross-sectional area $A = (5\text{ cm})^2 = 2.5 \times 10^{-3}\text{ m}^2$,

$$\mathscr{R} \simeq -1\,, \quad \mathscr{T} = 0\,.$$

- From Eq. (20), the radiation force for SPL of 200 dB re 1μPa is

$$F \simeq \frac{p_0^2 A}{\rho_1 c_1^2} = 0.0023\text{ N}.$$

- Wang and Lee[24] perform a more general analysis for $\ell = 0$:

$$\begin{aligned} p_\mathrm{i} &= A_\mathrm{i} J_\ell(k_r r)\exp[i(k_1 z + \ell\theta - \omega t)], \\ p_\mathrm{r} &= A_\mathrm{r} J_\ell(k_r r)\exp[-i(k_1 z - \ell\theta + \omega t)], \\ p_\mathrm{t} &= A_\mathrm{t} J_\ell(k_r r)\exp[i(k_2 z + \ell\theta - \omega t)]. \end{aligned}$$

  One could explore *vortex beam forces* ($\ell \neq 0$) at the interface.

[24] T. G. Wang and C. P. Lee. *Nonlinear Acoustics*. Ed. by M. F. Hamilton and D. T. Blackstock. 3rd. Cham, Switzerland: Springer, 2024. Chap. 6, Sec. 2.2.3.

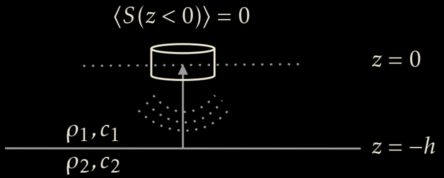
Radiation reaction: self-levitation

What SPL is required for a 1 kg speaker (5 cm cone radius) pointed at the (rigid) ground to levitate itself? Assume a paraxial beam.

- The ground is rigid, so $\mathscr{R} = 1$, $\mathscr{T} = 2$. The radiation force equals

$$F = \langle S(z<0)\rangle A = \frac{p_0^2 A}{2\rho_1 c_1^2}(1+\mathscr{R}^2). \tag{21}$$

- To counteract gravity, set $F = 9.8$ N. Inversion of Eq. (21) yields $p_0 = \sqrt{2F\rho_1 c_1^2/[A(1+\mathscr{R}^2)]}$, which corresponds to $176\,\text{dB re }20\,\mu\text{Pa}$.
- Recall that SPL = 120 dB re 20 μPa for a jet aircraft at takeoff.[25]
- For this p_0 in air ($\beta = 1.2$), a 440 Hz wave will shock at $(\beta\epsilon k)^{-1} = 1.16$ m.



[25] D. T. Blackstock. New York: Wiley, 2000, p. 52.

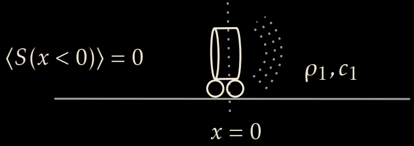
Radiation reaction: self-propulsion

Repeat the exercise for the same 1 kg speaker to reach 9.8 m/s in 1 s horizontally without friction.

- Now $\mathscr{R} = 0$, $\mathscr{T} = 0$, i.e., all waves are transferred to the medium.
- Again, $F = 9.8$ N. Thus

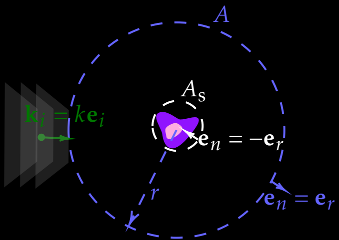
$$F = \langle S(x > 0)\rangle A = \frac{p_0^2 A}{2\rho_1 c_1^2}. \tag{22}$$

- Equation (22) is half the force given by Eq. (21).
- Inverting Eq. (22) yields $p_0 = \sqrt{2F\rho_1 c_1^2/A} \leftrightarrow 179\,\text{dB re } 20\,\mu\text{Pa}$.
- Only a 3 dB difference!



Calculation of radiation force in 3D

- The volume integral of $\nabla \cdot \langle \underline{\mathbf{S}} \rangle$ enclosing an object is the radiation force[26]



$$\mathbf{F} = \int_{V_\mathrm{s}} \nabla \cdot \langle \underline{\mathbf{S}} \rangle \, dV_\mathrm{s} = \oint_{A_\mathrm{s}} \langle \underline{\mathbf{S}} \rangle \cdot d\mathbf{A}_\mathrm{s}$$

P. J. Westervelt

- Invoking the far-field approximation and energy conservation yields

$$\boxed{\begin{aligned} F_\parallel &= \frac{p_0^2}{2\rho_0 c_0^2} \oint |\Phi|^2 (1 - \mathbf{e}_\mathrm{i} \cdot \mathbf{e}_r) \, d\Omega \,, && (23a) \\ F_\perp &= -\frac{p_0^2}{2\rho_0 c_0^2} \oint |\Phi|^2 \, \mathbf{e}_\mathrm{m} \cdot \mathbf{e}_r \, d\Omega \,, \quad d\Omega = r^{-2} \, dA \,. && (23b) \end{aligned}}$$

where Φ = scattered directivity, $\mathbf{e}_\mathrm{i} \cdot \mathbf{e}_r = \mathbf{k}_\mathrm{s} \cdot \mathbf{k}_\mathrm{i} / k^2$, and Ω = solid angle.

[26] P. J. Westervelt. *J. Acoust. Soc. Am.* 29 (1957), pp. 26–29.

Progressive wave radiation force on sphere

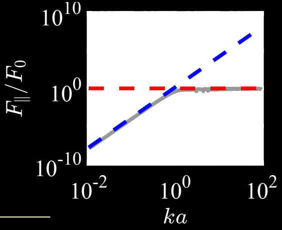
- Denoting $F_0 = p_0^2 \pi a^2 / 2\rho_0 c_0^2$, the exact radiation force is[27]

$$\frac{F_\parallel}{F_0} = \Re\left[\frac{4i}{(ka)^2}\sum_{n=0}^{\infty}\frac{n+1}{(2n+1)(2n+3)}(A_n^* + A_{n+1} + 2A_n^* A_{n+1})a_n^* a_{n+1}\right]. \quad (24)$$

- Denoting $f_1 = 1 - \beta_s/\beta_0$ and $f_2 = 2(\rho_s - \rho_0)/(2\rho_s + \rho_0)$, the $ka \ll 1$ force is[28]

$$F_\parallel = \frac{4\pi\langle I\rangle}{9c_0} a^2 (ka)^4 (f_1^2 + f_1 f_2 + \tfrac{3}{4} f_2^2), \quad (25)$$

- The $ka \gg 1$ limit of the radiation force on a rigid sphere is $F_\parallel = F_0$.



[27] T. S. Jerome, Yu. A. Ilinskii, E. A. Zabolotskaya, and M. F. Hamilton. *J. Acoust. Soc. Am.* 145 (2019), pp. 36–44.

[28] L. P. Gor'kov. *Sov. Phys. Dokl.* 6 (1962), pp. 773–775.

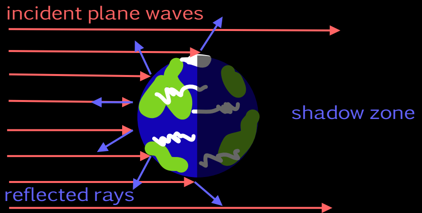
Radiation force exerted by Sun on Earth

- For $\langle I \rangle = 1361$ W/m^2 and $c = 3 \times 10^8$ m/s, the radiation pressure is

$$\langle P \rangle = \begin{cases} \langle I \rangle / c = 4.5 \times 10^{-6}\ \mathrm{N/m^2} & \text{perfect absorption} \\ 2\langle I \rangle / c = 9 \times 10^{-6}\ \mathrm{N/m^2} & \text{perfect reflection} \end{cases} \tag{26}$$

  which is $\sim 10^{11}$ smaller than atmospheric pressure (10^5 N/m^2).
- Radiation force on a perfectly absorptive Earth is 5.8×10^8 N, which is $\sim 10^{14}$ smaller than $F = Gm_1 m_2 / r^2 = 3.5 \times 10^{22}$ N.

  Had the pressure of sunlight exerted on the Viking spacecraft... been neglected, it would have missed Mars by about 15,000 km.[29]

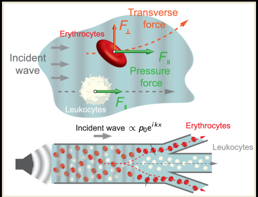


[29] E. Hecht. 5th. London: Pearson, 2017.

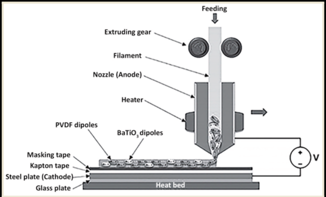
Radiation force in practice

Particle manipulation is relevant to biomedical engineering, advanced manufacturing, metamaterials, etc.

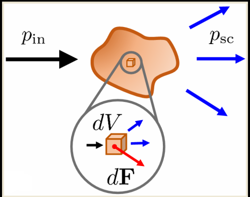
- Acoustic manipulation of biological materials
- Particle sorting for advanced manufacturing
- Analytical expressions of force due to progressive waves on asymmetric and/or inhomogeneous objects are unavailable



M. Smagin, I. Toftul, K. Y. Bliokh, and M. Petrov. *Phys. Rev. Appl.* 22 (2024), p. 064041



A. Smirnov et al. *Ceram. Int.* 47 (2021), pp. 10478–10511



T. S. Jerome. Ph.D. dissertation. University of Texas at Austin, 2022

Low-frequency approximation

- The Born and subwavelength approximations yield the scattered wave,

$$\tilde{p}_s(\mathbf{r}) = p_0 \frac{e^{ikr}}{r}\Phi, \quad \Phi(\mathbf{k}_s) = \frac{k^2}{4\pi}\left[\alpha_m + i\boldsymbol{\alpha}_c \cdot (\mathbf{e}_i - \mathbf{e}_r) + \alpha_d \mathbf{e}_i \cdot \mathbf{e}_r\right]. \tag{27}$$

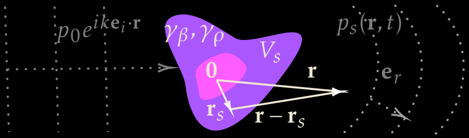
- The *acoustic polarizabilities* appearing in Eq. (27) are

$$\alpha_m = \int_{V_s} \gamma_\beta(\mathbf{r}_s)\, dV_s, \quad \alpha_d = \int_{V_s} \gamma_\rho(\mathbf{r}_s) dV_s, \tag{28a}$$

$$\boldsymbol{\alpha}_c = k\left[\int_{V_s} \gamma_\beta(\mathbf{r}_s)\mathbf{r}_s\, dV_s + \mathbf{e}_i \cdot \mathbf{e}_r \int_{V_s} \gamma_\rho(\mathbf{r}_s)\mathbf{r}_s\, dV_s\right], \tag{28b}$$

where $\gamma_\beta(\mathbf{r}) = \beta_s(\mathbf{r})/\beta_0 - 1$ and $\gamma_\rho(\mathbf{r}) = 1 - \rho_0/\rho_s(\mathbf{r})$.

- The force is $F_{\|} = p_0^2/(2\rho_0 c_0^2) \oint |\Phi(\mathbf{k}_s)|^2 (1 - \mathbf{e}_i \cdot \mathbf{e}_r) d\Omega_0$.[30]



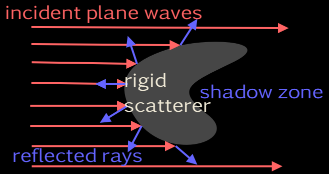
[30] P. J. Westervelt. *J. Acoust. Soc. Am.* 29 (1957), pp. 26–29.

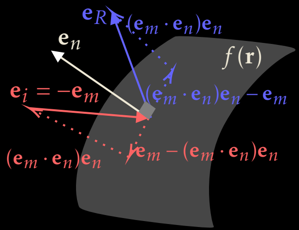
High-frequency approximation

- This is (to my knowledge) an unsolved problem.
- Recall that the radiation force in 3D is generally

$$\mathbf{F} = \oint_{A_s} \langle \underline{\mathbf{S}} \rangle \cdot \mathbf{e}_n \, dA_s \,, \tag{29}$$

- Westervelt's simplification of Eq. (29) to an integral in the far field does not apply to geometric acoustics.
- A new approach for radiation force must be derived for rays.





Outline

History

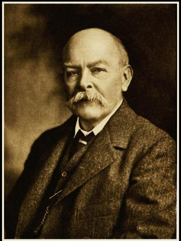
Waves on a string

Acoustic waves

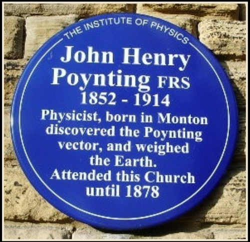
Conclusion

Poynting on radiation force

The Radiation Theory is only just starting on its journey. Its feet are not yet clogged by any certain data, and all directions are yet open to it. Any suggestion for its future course appears to be permissible, and it is only by trial that we shall find what ways are barred. At least we may be sure that it deals with real effects and that it must be taken into account.[31]



John Henry Poynting



A. P. Kapp. https://en.wikipedia.org/wiki/John_Henry_Poynting. CC-BY-SA-2.0

[31] J. H. Poynting. *Philos. Mag.* 9 (1905), pp. 393–406.

Conclusion

Summary

- Radiation force arises due to momentum conservation at $O(\epsilon^2)$.
- Radiation force is a locally nonlinear effect.
- Calculation of radiation force amounts to solving the linear scattering (or reflection-transmission) problem.

Unsolved problems related to radiation force

- …in geometric acoustics (my proposed postdoc)
- …due to multiple scattering
- …exerted by transients or multiple frequencies (now $\langle \partial g/\partial t \rangle \neq 0$)
- …on moving objects (v on the order of c)

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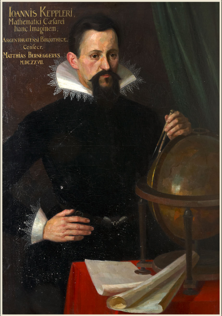
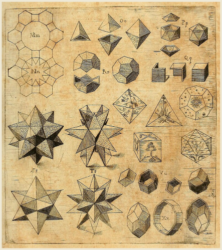
A sample of Johannes Kepler's legacy

- Elliptical orbits with the sun at the focus
- Planets sweep out equal areas in equal times
- $\tau^2 \propto a^3$
- *Musica universalis* (*Harmonice Mundi*)
- "There is a force in the earth which causes the moon to move"[a] (letter to Ferdinand II, HRE)
- Inverse square law of light (*Astronomiae Pars Optica*), foreshadowing Newton's gravitational inverse square law
- Optics and telescope design (correspondence with Galileo)
- Science fiction and interplanetary travel (*Somnium*)
- Music theory (abandoned Pythagorean tuning)

[a] M. Caspar. Courier Corporation, 2012.



Johannes Kepler



from *Harmonice Mundi*