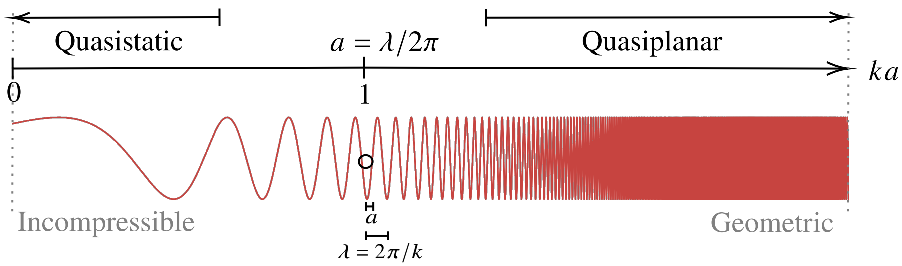
Synopsis of dissertation

Scattering and diffraction of acoustic waves in three problems with broken symmetry

The effects of asymmetry in acoustic **scattering** and **diffraction** phenomena are investigated across the frequency spectrum, which is characterized by the parameter ka, where k is the wavenumber and a is the characteristic size of the scatterer or source of sound:



Calculated first is the **acoustic radiation force** exerted by progressive waves on asymmetric scatterers in the Rayleigh limit ($ka \ll 1$). The Born approximation[1] in combination with Westervelt's far-field integral[2] leads to simple analytical expressions for the radiation force. Material asymmetry contributes to the force on the order of $(ka)^6$. For scatterers with symmetric material properties, Gor'kov's $O[(ka)^4]$ force[3] is recovered. The result informs efforts to acoustically transport small particles over large distances.

Next considered is the effect of low-ka asymmetric scattering in a **piezoelectric composite**. Constraints due to reciprocity and passivity are derived for the medium's constitutive relations, which fully couple continuum mechanics to electrodynamics. Limiting cases recover previous results that neglect magnetism,[4] piezoelectricity,[5] and elastodynamics.[6] The results guide the design of acoustic vector sensors.

Attention is finally turned to **vortex beams**, which are sound beams that break axisymmetry by an amount characterized by the orbital number ℓ.[7] The vortex field radiated by a circular piston is represented for all ka in terms of Bessel beams. Closed-form analytical solutions are derived in the paraxial approximation ($ka \gg 1$). In the limit that ka is infinite, the effects of diffraction are suppressed, leading to a geometric description of how the field depends on ℓ. In the absence of orbital motion ($\ell = 0$), the exact, paraxial, and ray solutions recover known descriptions of axisymmetric beams.[8] The analytical solutions and scaling laws derived simplify the modeling of vortex fields, which are used for particle manipulation and acoustic communications.

[1] P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, (McGraw-Hill, 1968), Eq. (8.1.20).
[2] P. J. Westervelt, *J. Acoust. Soc. Am.* **29**, 26–29 (1957), Eq. (2).
[3] L. P. Gor'kov, *Sov. Phys. Dokl.* **6**, 773-775 (1962), Eq. (10).
[4] R. Pernas-Salomón and G. Shmuel, *Phys. Rev. Appl.* **14**, 1–19 (2020), Eqs. (18), (20), (33), and (42).
[5] M. B. Muhlestein et al., *Proc. R. Soc. A* **472**, 1–15 (2016), Eqs. (3.10)–(3.17).
[6] J. A. Kong, *P. IEEE* **60**, 1036–1046 (1972), Eqs. (8) and (23).
[7] B. T. Hefner and P. L. Marston, *J. Acoust. Soc. Am.* 103, 2971 (1998).
[8] M. F. Hamilton, "Sound Beams," M. F. Hamilton and D. T. Blackstock, editors, *Nonlinear Acoustics, 3rd edition*, (Springer, 2024), Eqs. (8.19) and (8.37).